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Pre-Board Exam 2021-22 (Term-1)
CLASS- XII
SUB:- Mathematics(041)
Marking Scheme

Q.1. Sol. $\sin^{-1}(\sin 600) = \sin^{-1}\{\sin(540 + 60)\}$

$= \{\sin^{-1}\sin(3\pi + \frac{\pi}{3})\}$

$= \{\sin^{-1} - \sin\frac{\pi}{3}\}$

$= \{\sin^{-1}\sin(-\frac{\pi}{3})\}$

$= -\frac{\pi}{3}$ Ans. (b)

Q.2. Sol. Ans. (d)

Since, the value of function $f(x) = \tan x$ is $\pm\infty$(undefined) at $\forall\, x = (2n+1)\frac{\pi}{2}, n \in Z$.

Hence $f(x) = \tan x$ is discontinuous on the set $\{x : x = (2n+1)\frac{\pi}{2}, n \in Z\}$.

Q.3. Sol. Ans. (c)

Since P and Q are symmetric matrix

$\Rightarrow P^T = P$ and $Q^T = Q$

Now, $(PQ - QP)^T = (PQ)^T - (QP)^T$ [By properties]

$= Q^TP^T - P^TQ^T$ [By properties]

$= QP - PQ$ [Given]

$= -(PQ - QP)$

Hence, PQ − QP is a Skew Symmetric matrix.

Q.4. Sol. Ans. (d)

Obviously, the number of elements of matrix of order 3x3 is 9 and each entries can be field with 1 or 2 by 2 ways.

∴ No of such possible matrix = Total number of ways to fill all nine entries of such matrix

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^9 = 512$

Q.5. Sol. Ans. (d)

Here, $y = 2x^2 + 3\sin x$

$\Rightarrow \frac{dy}{dx} = 4x + 3\cos x$

$\Rightarrow \frac{dy}{dx}\Big]_{x=0}$ $=4x + 3\cos x = 4 \times 0 + 3\cos 0 = 3$

∴ Slop of tangent to the curve at (0, 1) = 3

⇒ Slop of normal to the curve at (0, 1) = $-\frac{1}{3}$

Q.6. Sol. Ans. ((b)

$|adjA| = |\text{A}|^{n-1}$, where n is order of matrix A.

Here n = 3, ∴ $|adjA| = |\text{A}|^{3-1} = |\text{A}|^2$

Q.7. Sol. Ans. (b)

$(1,1),(2,2),(3,3),(4,4) \in R$ ⇒ is reflexive

Also it is transitive.

But it is not symmetric as

$(1,3) \in R$ and $(3,1) \notin R$.

Q.8. Sol. Ans. (b)

Let $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements of matrix we get

$3x + y = 0$ - - - - - - - - - - - - (i)

$y - 2 = 5$ - - - - - - - - - - - - -(ii)

$y + 1 = 8$ - - - - - - - - - - - -- (iii)

$2 - 3x = 4$ - - - - - - - - - - - - -(iv)

So obtained equation (i) and (iv) are not consistent because

(i) ⇒$x = -\frac{1}{7}$and (iv) ⇒ $x = -\frac{2}{3}$

Therefore, the values of variables are not possible to find.

Q.9. Sol. Ans. (c)

Here, $y = e^{2x}$

$\Rightarrow \frac{dy}{dx} = 2e^{2x}$

$\Rightarrow \frac{dy}{dx}\Big]_{(0,1)} = 2 \times e^0 = 2 \times 1 = 2$

∴ Slop of tangent to the curve $y = e^{2x}$ at (0, 1) = 2

⇒ Equation of tangent to the curve $y = e^{2x}$ at (0, 1) is

$\frac{y-1}{x-0} = 2$

$\Rightarrow y = 2x + 1$

For meeting point with x - axis, putting y = 0 we get

$0 = 2x + 1$

$\Rightarrow x = -\frac{1}{2}$

Hence required point is $\left(-\frac{1}{2}, 0\right)$.

Q.10. Sol. Ans - (b)

$\tan^{-1}\sqrt{3} = \tan^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

$\sec^{-1}(-2) = \sec^{-1}\left(-\sec\frac{\pi}{3}\right)$

$\sec^{-1}\left(\sec\left(\pi - \frac{\pi}{3}\right)\right)$

$= \sec^{-1}\left(\sec\frac{2\pi}{3}\right)$

$= \frac{2\pi}{3}$

$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Q.11. (c) $R \subset A \times B$

Q.12. Ans. (c)

Sol. Given expression is

$x = e^{y+e^{y+\cdots\ldots\ldots to\ \infty}}$

$\Rightarrow \quad x = e^{y+x}$,

Taking log both sides, we have

$\log x = \log e^{y+x}$

$\Rightarrow \log x = y + x$

Differentiating, we get

$\frac{1}{x} = \frac{dy}{dx} + 1$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1$

$\Rightarrow \frac{dy}{dx} = \frac{1-x}{x}$

Q.13. Sol. Ans. (a)

PY+WY is defined ⇒ PY and WY are defined and order PY = order of WY.

PY is defined ⇒ No. of columns of P = No. of rows of Y

⇒ k = 3

Also order of PY = Order of WY ⇒ $p \times k = n \times k \Rightarrow p = n$

Therefore, PY+WY is defined if k = 3 and p = n.

Q.14. Sol. Ans. (c)

Let $y = \sin(\log x)$

$\Rightarrow \quad \frac{dy}{dx} = \cos(\log x).\frac{1}{x}$

Q.15. Sol. Ans. (c)

Let A = $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

⇒ k A = $\begin{bmatrix} ka_1 & ka_2 & ka_3 \\ kb_1 & kb_2 & kb_3 \\ kc_1 & kc_2 & kc_3 \end{bmatrix}$

⇒ $|kA| = \begin{vmatrix} ka_1 & ka_2 & ka_3 \\ kb_1 & kb_2 & kb_3 \\ kc_1 & kc_2 & kc_3 \end{vmatrix}$

Taking k common from R_1, R_2 and R_3, we get

$$\Rightarrow |\mathrm{kA}| = k^3 \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Rightarrow |\mathrm{kA}| = k^3|\mathrm{A}|$$

Q.16. Sol. Ans. (c).

Given curves are

$$x^3 - 3xy^2 + 2 = 0 \text{ - - - - - - - - - - - - - (i)}$$

$$\text{and } 3x^2y - y = 2 \text{ - - - - - - - - - - - - - (ii)}$$

If m_1 and m_2 are the slop of tangents at intersecting point of both curve (i) and (ii) respectively, then

$$m_1 = \frac{dy}{dx} \text{ for curve (i)}$$

$$\Rightarrow m_1 = \frac{x^2-y^2}{2xy} \text{ - - - - - - - - - - -(iii)}$$

$$\text{Also, } m_2 = \frac{dy}{dx} \text{ for curve (ii)}$$

$$\Rightarrow m_2 = \frac{-2xy}{x^2-y^2} \text{ - - - - - - - - - - - (iv)}$$

Multiplying (iii) and (iv), we get

$$m_1 \cdot m_2 = \frac{x^2-y^2}{2xy} \cdot \frac{-2xy}{x^2-y^2}$$

$$\Rightarrow m_1 \cdot m_2 = -1$$

⇒ Both curve cut at right angle.

Q.17. Sol. Ans. (a)

Here A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$= I(identity matrix)

Q.18. Sol. Ans. (d)

Here, $e^y(x+1) = 1$

$$\Rightarrow \quad e^y = \frac{1}{(x+1)}$$

$$\Rightarrow \quad \log e^y = \log \frac{1}{(x+1)}$$

$$\Rightarrow \quad y = -\log(x+1)$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{1}{(x+1)}$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} = \left(\frac{dy}{dx}\right)^2$$

Q.19. Ans. (c)

Sol.

Corner points	$z = 22x + 18y$
O(0, 0)	0
A(16, 0)	352
C(8, 12)	392
B(0, 20)	360

From table, we get z is maximum at point C (8, 12).

Q.20. Ans. (c)

$f(x) = x^2 - x + 1$

$\Rightarrow \quad f'(x) = 2x - 1$

Obviously, $\Rightarrow \quad f'(x) < 0$ for $x \in \left(-1, \frac{1}{2}\right)$, i.e. $f(x)$ is decreasing.

And $\quad f'(x) > 0$ for $x \in \left(\frac{1}{2}, 1\right)$, i.e. $f(x)$ is increasing.

GROUP B

Q.21. Sol. Ans. (d)

f is not one-one because

$f(-2) = (-2)^4 = 16$

$f(2) = (2)^4 = 16$

i.e. -2 and 2 $\in R$ (Domain) have same f-image in R (co-domain)

$\Rightarrow f$ is not one-one.

Also $f(x) = x^4$ never achieve negative value.

$\Rightarrow$ All negative real number of Co-domain R have no pre-image in Domain R.

$\Rightarrow f$ is not onto.

Hence f is neither one-one nor onto.

Q.22. Sol. Ans. (c)

Here, $x = a\cos^3\theta$

Differentiating both sides w.r.t. θ we get

$\frac{dx}{d\theta} = 3a\cos^2\theta\,(-\sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = -3a\cos^2\theta \cdot \sin\theta$ - - - - - - - - - - - - - - - - (i)

Also, $y = a\sin^3\theta$

Differentiating both sides w.r.t. θ we get

$\frac{dy}{d\theta} = 3a\sin^2\theta \cdot \cos\theta$ - - - - - - - - - - - - - - - - (ii)

$\because \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3a\sin^2\theta \cdot \cos\theta}{-3a\cos^2\theta \cdot \sin\theta}$ [From (i) and (ii)]

$= \frac{\sin\theta}{-\cos\theta}$

$\Rightarrow \frac{dy}{dx} = -\tan\theta$

$\Rightarrow \frac{dy}{dx}$ at $\theta = \frac{\pi}{4} = -1$

Q.23. Sol. Ans. (c)

Now, the value Z is evaluated at corner points as

Corner points	$z = 12x + 16y$
O(0, 0)	0
A(600, 0)	7200

B(1050, 150)	15000
C(800, 400)	16000

From table, we get z is maximum at point C (800, 400).

Q.24. Sol. Ans. (d)

Let y = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and z = $\tan^{-1} x$.

Here, we have to find out $\frac{dy}{dz}$.

Now, y = $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

Let $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

Now, y = $\tan^{-1}\left[\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\sec\theta-1}{\tan\theta}\right]$

$= \tan^{-1}\left[\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right]$

$= \tan^{-1}\left[\frac{1-\cos\theta}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= \tan^{-1}\left[\frac{2\sin^2{\theta}/{2}}{2\sin{\theta}/{2}\cos{\theta}/{2}}\right]$

$= \tan^{-1}\left[\frac{\sin{\theta}/{2}}{\cos{\theta}/{2}}\right]$

$= \tan^{-1}\left[\tan{\theta}/{2}\right]$

y = ${\theta}/{2}$

y = $\frac{1}{2}\tan^{-1} x$

$\left(\begin{array}{l} \because -\infty < x < \infty \\ \Rightarrow \tan(-\frac{\pi}{2}) < \tan\theta < \tan\frac{\pi}{2} \Rightarrow \\ -\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{4} \\ \Rightarrow \frac{\theta}{2} \in \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \subset \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \end{array}\right)$

Differentiating both sides w.r.t. x, we get

$\frac{dy}{dx} = \frac{1}{2(1+x^2)}$ - - - - - - - - - - - - - - - - (i)

Also, z = $\tan^{-1} x$

Differentiating both sides w.r.t. x, we get

$\frac{dz}{dx} = \frac{1}{1+x^2}$ - - - - - - - - - - - - - - - - (ii)

$\because \frac{dy}{dz} = \frac{{dy}/{dx}}{{dz}/{dx}}$

$= \frac{1}{2(1+x^2)} \times \frac{1+x^2}{1}$

$\Rightarrow \frac{dy}{dz} = \frac{1}{2}$

Q.25. Sol. Ans. (a)

$[1+2x+15 \quad 3+5x+3 \quad 2+x+2]\begin{bmatrix}1\\2\\x\end{bmatrix} = [0]$

$\Rightarrow [2x+16 \quad 5x+6 \quad x+4]\begin{bmatrix}1\\2\\x\end{bmatrix} = [0]$

$\Rightarrow [2x+16+10x+12+x^2+4x] = [0]$

$\Rightarrow [16x+28+x^2] = [0]$

Equating the corresponding elements, we get

$x^2+16x+28=0$

$\Rightarrow x^2+16x+28 = 0$

$\Rightarrow x^2+14x+2x+28 = 0$

$\Rightarrow x(x+14)+2(x+14) = 0$

$\Rightarrow (x+14)(x+2) = 0$

$\Rightarrow x = -14, -2$

Q.26. Sol. Ans. (c)

Here, $f(x) = 2x^2 - 3x$

$\Rightarrow f'(x) = 4x - 3$

Now, $f'(x) = 0$

$\Rightarrow 4x - 3 = 0$

$\Rightarrow x = \frac{3}{4}$ is critical point.

The critical point $x = \frac{3}{4}$ divide the domain of $f(x)$ i.e. R into two open intervals as - $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.

For, $\left(-\infty, \frac{3}{4}\right)$

$f'(x) = 4x - 3$

$f'(x)]_{x=0} = 4\times 0 - 3 = -3 < 0 \qquad \left[\because 0 \in \left(-\infty, \frac{3}{4}\right)\right]$

$\Rightarrow f'(x) < 0$ on interval $\left(-\infty, \frac{3}{4}\right)$

Hence, $f(x)$ is strictly decreasing on $\left(-\infty, \frac{3}{4}\right)$

For, $\left(\frac{3}{4}, \infty\right)$

$f'(x) = 4x - 3$

$f'(x)]_{x=1} = 4\times 1 - 3 = 1 > 0 \qquad \left[\because 1 \in \left(\frac{3}{4}, \infty\right)\right]$

$\Rightarrow f'(x) > 0$ on interval $\left(\frac{3}{4}, \infty\right)$

Hence, $f(x)$ is strictly increasing on $\left(\frac{3}{4}, \infty\right)$

Therefore, Strictly increasing on $\left(\frac{3}{4}, \infty\right)$, Strictly decreasing on $\left(-\infty, \frac{3}{4}\right)$

Q.27. Sol. Ans. (b)

$= \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$= \cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \times \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right)$

$= \cot^{-1}\left(\frac{2+2\cos x}{2\sin x}\right)$

$= \cot^{-1}\left(\frac{1+\cos x}{\sin x}\right)$

$= \cot^{-1}\left(\frac{2\cos^2(x/2)}{2\sin x/2\cos x/2}\right)$

$= \cot^{-1}\left(\frac{\cos x/2}{\sin x/2}\right)$

$= \cot^{-1}(\cot x/2)$

$= \frac{x}{2}$ $\left[\because\ 0 < x < \frac{\pi}{4} \Rightarrow 0 < \frac{x}{2} < \frac{\pi}{8} \Rightarrow \frac{x}{2} \in \left(0, \frac{\pi}{8}\right) \subset \left(0, \frac{\pi}{2}\right)\right]$

Q.28. Sol. Ans. (a)

Now 2A = $2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 8 - 32 = -24$

Also, $4|A| = 4\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 4\times\{1 \times 2 - 2 \times 4\} = 4 \times (-6) = -24$

Hence, $|2A| = 4|A|$

Q,29. Sol. Ans. (b)

Obviously, for f to be maximum, $4x^2 + 2x + 1$ should be minimum i.e.

For minimum value of $4x^2 + 2x + 1$

Let $y = 4x^2 + 2x + 1$

$\Rightarrow \frac{dy}{dx} = 8x + 2$

For extremum value of y

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow 8x + 2 = 0$

$\Rightarrow x = -\frac{1}{4}$

Now, $\frac{d^2y}{dx^2} = 8$

$\Rightarrow \left.\frac{d^2S}{dx^2}\right]_{x=-\frac{1}{4}} = +ve$

$\Rightarrow$ For $x = -\frac{1}{4}$, $4x^2 + 2x + 1$ is minimum.

$\Rightarrow$ For $x = -\frac{1}{4}$, f is maximum.

$\therefore$ Required maximum value = $\frac{1}{4\left(-\frac{1}{4}\right)^2+2\left(-\frac{1}{4}\right)+1} = \frac{1}{\frac{1}{4}-\frac{1}{2}+1} = \frac{4}{3}$

Q.30. Sol. Ans. (b)

Here, only one pair $(8, 4)$ follow the condition that $a = 2b$, where $a = 8$ and $b = 4$.

Q.31. Sol. Ans. (c)

At $x = 1$

RHD = $\lim_{h\to 0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim_{h\to 0} \dfrac{|1+h-1| - |1-1|}{h}$

$= \lim_{h\to 0} \dfrac{h}{h}$ $\quad [\because |h| = h, |0| = 0]$

$= \lim_{h\to 0} 1$

RHD = 1 - - - - - - - - - - - - - - (i)

LHD = $\lim_{h\to 0} \dfrac{f(1-h) - f(1)}{-h}$

$= \lim_{h\to 0} \dfrac{|1-h-1| - |1-1|}{-h}$

$= \lim_{h\to 0} \dfrac{|-h|}{-h}$

$= \lim_{h\to 0} \dfrac{h}{-h}$ $\quad [\because |h| = h]$

$= \lim_{h\to 0} (-1)$

LHD = -1 - - - - - - - - - - - - - - - - (ii)

(i) and (ii) ⇒ RHD ≠ LHD at $x = 1$.

Hence $f(x)$ is not differentiable at $x = 1$.

Q.32. Sol. (c)

$\because A^2 = A \times A$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \\ 1+1+1 & 1+1+1 & 1+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

= 3 A

Q.33. Sol. Ans. (c)

Here, Objective function is

Z = $3x + 2y$ …………….. (i)

And constraints are

$x + 2y \leq 10$ …………….. (ii)

$3x + y \leq 15$ …………….. (iii)

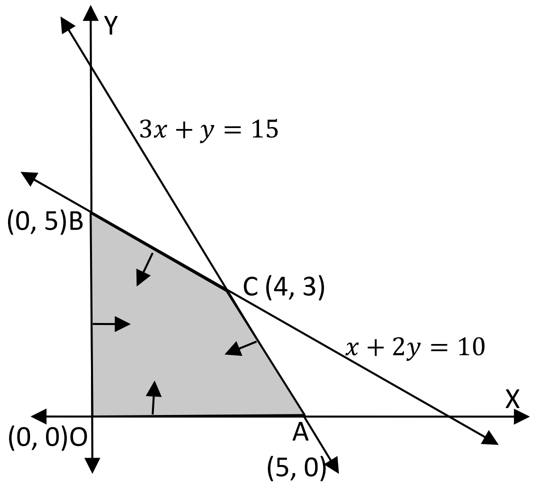
$x \geq 0$ …………….. (iv)

$y \geq 0$ …………….. (v)

On plotting graph of above constraints (or inequalities)(ii), (iii), (iv) and (v) we get bounded shaded region as feasible region having corner points O, A, C and B.

The coordinates of the corner-points of the feasible region OACB are O (0, 0), A (5, 0), C (4, 3) and B (0, 5). These points are obtained by solving the corresponding intersecting lines.

Now, the value of Z is evaluated at corner point as



Corner points	$z = 3x + 2y$
O(0, 0)	0
A(5, 0)	15
C(4, 3)	18
B(0, 5)	10

The maximum value of Z is 18 at $x = 4, y = 3$.

Q.34. Sol. Ans. (b)

Here, $f(x) = 2x^3 - 3x^2 - 36x + 7$

$\Rightarrow f'(x) = 6x^2 - 6x - 36$

Now, $f'(x) = 0$

$\Rightarrow 6x^2 - 6x - 36 = 0$

$\Rightarrow 6(x^2 - x - 6) = 0$

$\Rightarrow 6\{x^2 - 3x + 2x - 6\} = 0$

$\Rightarrow 6\{x(x - 3) + 2(x - 3)\} = 0$

$\Rightarrow 6(x - 3)(x + 2) = 0$

$\Rightarrow x = -2, 3$ are critical points.

The critical points $x = -2, 3$ divide the domain of $f(x)$ i.e. R into three disjoint open intervals $(-\infty, -2), (-2, 3)$ and $(3, \infty)$.

For, $(-\infty, -\mathbf{2})$

$f'(x) = 6(x - 3)(x + 2)$

$f'(x)]_{x=-3} = +ve \times -ve \times -ve = +ve$ $\quad [\because -3 \in (-\infty, -2)]$

$\Rightarrow f'(x) > 0$ on interval $(-\infty, -2)$

Hence, $f(x)$ is strictly increasing on $(-\infty, -2)$

For, $(-\mathbf{2}, \mathbf{3})$

$f'(x) = 6(x - 3)(x + 2)$

$f'(x)]_{x=1} = +ve \times -ve \times +ve = -ve$ $\quad [\because 1 \in (-2, 3)]$

$\Rightarrow f'(x) < 0$ on interval $(-2, 3)$

Hence, $f(x)$ is strictly decreasing on $(-2, 3)$

For, $(\mathbf{3}, \infty)$

$f'(x) = 6(x-3)(x+2)$

$f'(x)]_{x=4}$ = $+ve \times +ve \times +ve \ = +ve$ $\quad$ $[\because 4 \in (3, \infty)]$

$\Rightarrow f'(x) > 0$ on interval $(3, \infty)$

Hence, $f(x)$ is strictly increasing on $(3, \infty)$

Therefore, (a) Strictly increasing on $(-\infty, -2)$ and $(3, \infty)$.

(b) Strictly decreasing on $(-2, 3)$.

Q.35. Sol. Ans. (c)

$\because A^2 = I$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} . \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements of matrix we get

$\alpha^2 + \beta\gamma = 1$

$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$

Q.36. Sol. Let, $\sin^{-1}(.8) = \theta \Rightarrow \sin\theta = 0.8$

$\Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1 - (.8)^2}$

$\Rightarrow \cos\theta = \sqrt{1 - .64}$

$\Rightarrow \cos\theta = \sqrt{.36}$

$\Rightarrow \cos\theta = .6$

$\therefore \sin(2\sin^{-1}(.8)) = \sin 2\theta = 2\sin\theta\cos\theta = 2 \times .8 \times .6 = .96$

Ans. (c)

Q.37. Sol. Ans. (b)

Let x be te pre image of 5

$\therefore f^{-1}(5) = x \Rightarrow f(x) = 5$

$\Rightarrow x^2 + 1 = 5$

$\Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

i.e. Pre-image of 5 is -2, +2.

Q.38. Sol. Ans. (b)

Here, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

From questions, $A + A^T = I$

$$\Rightarrow \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Equating the corresponding elements of matrix we get

$2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = \cos\frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{3}$

Q.39. Sol. Ans. (b)

Given, $y = x^2 + ax + 25$ - - - - - - -(i)

$\Rightarrow \frac{dy}{dx} = 2x + a$

The curve (i) touches the x - axis ⇒ x – axis is tangent to curve at meeting point.

$\Rightarrow \frac{dy}{dx} = 0$

$\Rightarrow 2x + a = 0$

$\Rightarrow x = -\frac{a}{2}$

⇒ The co-ordinate of meeting point is $\left(-\frac{a}{2}, 0\right)$, therefore it satisfy the curve (i)

$\Rightarrow \left(-\frac{a}{2}\right)^2 + a\left(-\frac{a}{2}\right) + 25 = 0$

$\Rightarrow \frac{a^2}{4} - \frac{a^2}{2} + 25 = 0$

$\Rightarrow -a^2 + 100 = 0$

$\Rightarrow a = \pm 10$

Q.40. Sol. Ans. (a)

Here A = $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = I(identity matrix)

SECTION C

Q.41. Sol. Ans. (c)

Since, Z is maximum at (120, 0) and (60, 30)

$\Rightarrow \quad 120a = 60a + 30b$

$\Rightarrow \quad 120a - 60a - 30b = 0$

$\Rightarrow \quad 60a - 30b = 0$

$\Rightarrow \quad 2a - b = 0$

Q.42. Sol. Ans. (a)

Let the required point to the curve $y^2 = 4x$ be (x_0, y_0) at which the tangent is $y = x + 1$.

⇒ Slop of tangent to given curve $[y^2 = 4x]$ at (x_0, y_0) = slop of line $y = x + 1$

$\Rightarrow \left[\frac{dy}{dx}\right]_{(x_0, y_0)} = 1$ - -- - - - - - - - - - - - - - - (i)

Now, given curve is $y^2 = 4x$

Differentiating w.r.t. x we get

$2y\frac{dy}{dx} = 4$

$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$

$\Rightarrow \left[\frac{dy}{dx}\right]_{(x_0,y_0)} = \frac{2}{y_0}$ - - - - - - - - - - - - - (ii)

From (i) and (ii), we get

$\Rightarrow \frac{2}{y_0} = 1$

$\Rightarrow y_0 = 2$

Since, (x_0, y_0) also lie on line $y = x + 1$

$\Rightarrow y_0 = x_0 + 1$

$\Rightarrow 2 = x_0 + 1$

$\Rightarrow x_0 = 1$

Hence required points are (1, 2).

Q.43. Sol. Ans. (a)

Here, $f(x) = x^3 - 3x^2 + 3x - 100$

$\Rightarrow f'(x) = 3x^2 - 6x + 3$

$\Rightarrow f'(x) = 3(x^2 - 2x + 1)$

$\Rightarrow f'(x) = 3(x - 1)^2$

$\Rightarrow f'(x) = +ve \times +ve = +ve$

$\Rightarrow f'(x) > 0, \ \forall x \in R$

Hence, $f(x)$is strictly increasing in R.

Q.44. Sol. Ans. (b)

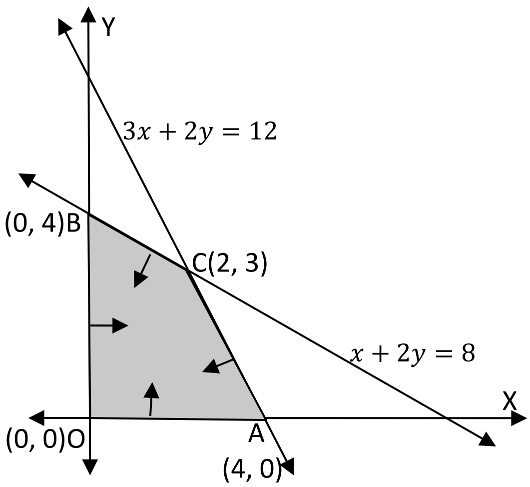
Feasible region of given constraints is

$x + 2y \leq 8$ …………….. (ii)

$3x + 2y \leq 12$ …………….. (iii)

$x \geq 0$ …………….. (iv)

$y \geq 0$ …………….. (v)



Q.45. Sol. Ans. (b)

$x^2 - 36 = 36 - 36$

$\Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36$

$\Rightarrow x^2 = \pm 6$

Q.46. Sol. Ans. (c)

Let length and width of window be x and y respectively. If A be the area of opening of window, which admit light then

Perimeter of window = $x + 2y + \pi \cdot \frac{x}{2}$

$\Rightarrow \quad x + 2y + \pi \cdot \frac{x}{2} = 10$

$\Rightarrow \quad 2x + 4y + \pi x = 20$

Q.47. Sol. Ans. (d)

$$A = x \cdot y + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$$

$$\Rightarrow A = xy + \frac{\pi x^2}{8}$$

$$\Rightarrow A = x \cdot \left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$\Rightarrow A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8}$$

$$\Rightarrow A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

Q.48. Sol. Ans. (a)

$\because \quad A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$

$$\Rightarrow \frac{dA}{dx} = 5 - x - \frac{\pi x}{4}$$

For extremum value of A

$$\Rightarrow \frac{dA}{dx} = 0$$

$$\Rightarrow 5 - x - \frac{\pi x}{4} = 0$$

$$\Rightarrow x + \frac{\pi x}{4} = 5$$

$$\Rightarrow 4x + \pi x = 20$$

$$\Rightarrow x(4 + \pi) = 20$$

$$\Rightarrow x = \frac{20}{4+\pi}$$

Now, $\frac{d^2A}{dx} = -1 - \frac{\pi}{4}$

$$\Rightarrow \left.\frac{d^2A}{dx^2}\right]_{x=\frac{20}{4+\pi}} = -ve$$

Hence for maximum value of A, $x = \frac{20}{4+\pi}$ m.

Q.49. Sol. Ans. (a)

$\because \quad 2x + 4y + \pi x = 20$

$\Rightarrow \quad 4y = 20 - 2x - \frac{20\pi}{4+\pi}$

$\Rightarrow \quad 4y = 20 - \frac{40}{4+\pi} - \frac{20\pi}{4+\pi}$

$\Rightarrow \quad y = 5 - \frac{10}{4+\pi} - \frac{5\pi}{(4+\pi)}$

$\Rightarrow y = 5 - \frac{10}{4+\pi} - \frac{5\pi}{(4+\pi)}$

$\Rightarrow y = \frac{20+5\pi-10-5\pi}{(4+\pi)}$

$\Rightarrow y = \frac{10}{(4+\pi)}$ m

Q.50. Sol. Ans. (d)

Area of window = ar (rectangular part) + ar (semicircular part)

$$= \frac{20}{4+\pi} \cdot \frac{10}{4+\pi} + \frac{1}{2}\pi \cdot \left(\frac{20}{2(4+\pi)}\right)^2$$

$$= \frac{20}{4+\pi} \cdot \frac{10}{4+\pi} + \frac{\pi}{8} \cdot \frac{400}{(4+\pi)^2}$$

$$= \frac{200}{(4+\pi)^2} + \frac{50\pi}{(4+\pi)^2}$$

$$= \frac{200+50\pi}{(4+\pi)^2} \text{ sq. m}$$