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PRE BOARD II EXAMINATION (2022-23)
(SET-I)

SUBJECT: MATHEMATICS (041) **MAX. MARKS : 80**
CLASS : XII **DURATION: 3 HRS**

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's** and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)**-type questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)**-type questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)**-type questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

SECTION – A

Questions 1 to 20 carry 1 mark each.

1. If A = $\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}$ and A + A' = I , then the value of α is :-

(a) π/3 (b) π/6 (c) π (d) 3π/2

2. If A = $\begin{bmatrix} 2x & 6 \\ -1 & 1 \end{bmatrix}$ is a singular matrix , then the value of x is :-

(a) x = -3 (b) x = 3 (c) x = 1 (d) x = -2

3. If $(\hat{i}+3\hat{j}+8\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=0$ then λ and μ are respectively:

(a) 27, -9 (b) 9, 9 (c) -9, 24 (d) -1, 1

4. If $f(x)=x^2 sin\frac{1}{x}$, where x ≠ 0, then the value of the function f at x = 0, so that the function is continuous at x = 0, is

(a) 1 (b) -1 (c) 0 (d) None of these

5. The value of $\int_0^a \frac{\sqrt{a}}{\sqrt{x}+\sqrt{a-x}}dx$ is:

(a) 0 (b) a (c) a^2 (d) a/2

6. The order and the degree of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ are:

(a) 1, 1 (b) 2, 1 (c) 1, 2 (d) 3, 1

7. Corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let F =4x + 6y be the objective function. The minimum value of F occurs at

(a) Only (0, 2)
(b) Only (3, 0)
(c) the mid-point of the line segment joining the points (0, 2) and (3, 0)
(d) any point on the line segment joining the points (0, 2) and (3, 0)

8. The projection of the vector $2\hat{i}+3j+2k$ *on the vector* $\hat{i}+2j+k$ is

(a) 5/√6 (b) 10/√3 (c) 10/√6 (d) 5/√3

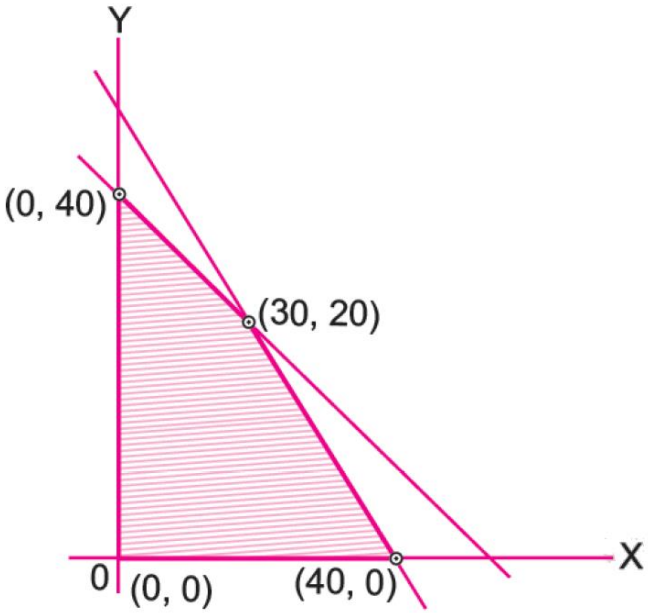
9. The value of $\int \frac{1}{e^x - 1} dx$ is:

(a) log e^x + C (b) log|1 – e^{-x}| + C (c) log log $\frac{1}{e^x}$ + C (d) log|e^x – 1| + C

10. The cofactor of (-1) in the matrix $\begin{bmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ is:

(a) 1 (b) 2 (c) -1 (d) 0

11. Feasible region (shaded) for a LPP is shown in the given figure.
The maximum value of the Z = 0.4x + y is



(a) 45 (b) 40 (c) 50 (d) 41

12. For what value of k ∈ N, $\begin{vmatrix} k & 3 \\ 4 & k \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ 0 & 1 \end{vmatrix}$ is .

(a) 0 (b) 1 (c) 3 (d) 4

13. If *A* is a square matrix of order 3, |A'| = -3 , then |AA'| is equal to:-

(a) 9 (b) -9 (c) 3 (d) -3

14. If A and B are two events such that P(A) = 1/2, P(B) = 1/3 and P(A/B) = 1/4, then P(A' ∩ B') equals

(a) 1/12 (b) 3/4 (c) 1/4 (d) 3/16

15. If $\frac{dy}{dx} = y \sin 2x$, y(0) = 1, then solution is

(a) y = $e^{\sin^2 x}$ (b) y = $\sin^2$x (c) y = $\cos^2$x (d) y = $e^{\cos^2 x}$

16. If y = 5 cos x – 3 sin x, then $\frac{d^2y}{dx^2}$ is equal to:

(a) 9y (b) y (c) 25y (d) - y

17. The value of λ such that the vector $\vec{a} = 2\hat{i} + \lambda\hat{j} + k$ and $\vec{b} = \hat{i} + 2\hat{j} + 3k$ are orthogonal is:

(a) 3/2 (b) −1/2 (c) −5/2 (d) 1/2

18. If a line makes angles α, β, γ with the positive direction of co-ordinates axes, then find the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$.

(a) 1 (b) 2 (c) 3 (d) 4

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion: If the cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$, then its vector form is $\vec{r} = 5\hat{i} - 4j + 6k + \lambda(3\hat{i} + 7j + 2k)$

Reason: The cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x-3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is $\frac{x+3}{-2} = \frac{y-4}{4} = \frac{z+8}{-5}$.

20. Assertion (A): $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$

Reason (R): $\sin^{-1}(\sin\theta) = \theta$, if $\theta \in [-\pi/2, \pi/2]$

SECTION – B

Questions 21 to 25 carry 2 marks each.

21. Let f: R→R be defined by $f(x) = x^3 + 1$. State whether f(x) is one-one or not .

OR

Evaluate $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right]$

22. Find the relationship between 'a' and 'b' so that the function f(x) defined by

$$f(x) = \begin{cases} ax + 1, if\ x \le 3 \\ bx + 3, if\ x > 3 \end{cases} \text{ is continuous at } x = 3$$

23. If $|\vec{a} \times \vec{b}|^2 + (\vec{a}.\vec{b})^2 = 144$ and $|\vec{a}| = 4$, then find the value of $|\vec{b}|$.

24. If $\hat{a}$ and $\hat{b}$ are unit vectors and θ is the angle between them, then prove that $\cos\frac{\theta}{2} = \frac{1}{2}|\hat{a} + \hat{b}|$

OR

Find the equation of a line parallel to x – axis and passing through origin.

25. The volume of a spherical balloon is increasing at the rate of 25 cm^3/sec. Find the rate of change of its surface area at the instant when its radius is 5cm.

SECTION – C

Questions 26 to 31 carry 3 marks each.

26. A die is thrown twice and the sum of the numbers appearing is observed to be 6.What is the conditional probability that the number 4 has appeared at least once ?

OR

Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards.What is the probability that first two cards are kings and the third card is an ace?

27. Evaluate: $\int_0^{\pi} \frac{x}{1+sinx} dx$

OR

Evaluate: $\int_1^3 |x^2 - 2x| \, dx.$

28. Evaluate $\int \frac{2x}{(x^2+2)(x^2+3)} \, dx$

29. Find the general solution of the following differential equation; x dy – (y + $2x^2$)dx = 0

OR

Solve the differential equation: $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$; given y = 0 when x = 0.

30. Solve the following Linear Programming Problem graphically:
Maximise Z = x + 2y subject to the constraints: x + 2y ≥ 100; 2x – y < 0; 2x + y ≤ 200; x, y ≥ 0

31. Evaluate: $\int e^x \frac{(1-\sin x)}{(1-\cos x)} dx$

SECTION – D

Questions 32 to 35 carry 5 marks each.

32. Using integration, find the area of a ΔABC, the coordinates of vertices being A(-1, 0), B(1, 3), and C(3,2).

33. Find the shortest distance between the lines $\vec{r} = 3\hat{i} + 2j - 4k + \lambda(\hat{i} + 2j + 2k)$ and $\vec{r} = 5\hat{i} - 2j + \mu(3\hat{i} + 2j + 6k)$. If the lines intersect find their point of intersection.

OR

Find the coordinates of the point where the line through the points A(3, 4 ,1) and B(5,1,6) crosses the XY-plane.

34. Show that the relation R is the set A = $\{x \in Z, 0 \leq x \leq 12\}$ given by

R = $\{(a, b): a, b \in Z, |a - b| \text{ is a multiple of } 3\}$ is an equivalence relation.

Find the set of all elements related to 1.

OR

Show that the relation S in the set R of real numbers, defined as
S = {(a, b) : a, b ∈ R and a ≤ b^3} is neither reflexive, nor symmetric, nor transitive.

35. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi vertical angle α is one-third that of the cone and the greatest volume of cylinder is $4/27\ \pi h^3 \tan^2\alpha$.

SECTION – E(Case Study Based Questions)

Questions 36 to 38 carry 4 marks each.

36. **Case-Study 1:** Two trusts A and B receive Rs.70000 and Rs.55000 respectively from central government to award prize to persons of a district in three fields agriculture , education and social service. Trust A awarded 10,5 and 15 persons in the field of agriculture ,education and social service respectively while trust B awarded 15,10 and 5 persons in the field of agriculture, education and social service respectively. The cost of all the prizes together amounts to Rs. 6000. Based on the given information answer the following questions.

(i) If x , y , z are the amounts of each prize respectively and AX=B is the matrix form of the given information , Find adjoint of A.
(ii) Using inverse of matrix A find the values of x ,y and z.

37. **Case-Study 2:**

As we know good planning can save energy, time, and money. A farmer wants to construct a circular well and a square garden in his field. He wants to keep their perimeters 600 m.

Based on the above information, answer the following questions:
(i) If the radius of the circular garden is ‘r’ m and the side of the square garden is ‘x’ m, then what is the sum of their areas? And find the number which exceeds its square by the greatest possible.
(ii) At what radius, is the sum of their areas is least?

38. <u>Case-Study 3</u>: Read the following passage and answer the questions given below.

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There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.
(i) What is the probability that the shell fired from exactly one of them hit the plane?
(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?