NAVODAYA VIDYALAYA SAMITI
PRE-BOARD II EXAMINATION- 2023 – 24
CLASS: XII
MATHEMATICS

Time Allowed: 3 hours Max. Marks 80

General Instructions:
1. This question paper contains-five sections, A,B,C,D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)- type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 mark each.
5. Section D has 4 Long Answer (LA)-type questions of 5 mark each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

1. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A=B^2$, Then x equals :

(A) ± 1 (B) (-1) (C) 1 (D) 2

2. Let matrix $X=[x_{ij}]$ is given by $X=\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}$ then the matrix $Y=[m_{ij}]$ where m_{ij} = minor of x_{ij} is :

(A) $\begin{bmatrix}7 & -5 & -3\\ 19 & 1 & -11\\ -11 & 1 & 7\end{bmatrix}$ (B) $\begin{bmatrix}7 & -19 & 11\\ 5 & -1 & -1\\ 3 & 11 & 7\end{bmatrix}$ (C) $\begin{bmatrix}7 & 19 & -11\\ -3 & 11 & 7\\ -2 & -1 & -1\end{bmatrix}$ (D) $\begin{bmatrix}7 & 19 & -11\\ -1 & -1 & 1\\ -3 & -11 & 7\end{bmatrix}$

3. If A = [2 -3 4] B = $\begin{bmatrix}3\\ 2\\ 2\end{bmatrix}$, X = [1 2 3] and Y = $\begin{bmatrix}2\\ 3\\ 4\end{bmatrix}$, then AB+XY equals:

(A) [28] (B) [24] (C) 28 (D) 24

4. If x = A cos 4t + B sin 4t, then $\frac{d^2x}{dt^2}$ is equal to:

(A) x (B) - x (C) 16 x (D) -16 x

5. The differential equation of the form $\frac{dy}{dx}=f\left(\frac{y}{x}\right)$ is called :

(A) linear differential equation (B) partial differential equation
(C) homogeneous differential equation (D) non-homogenous differential equation

6. The value of determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is

(A) 10 (B) 8 (C) 7 (D) -7

7. The direction cosines of the line passing through the following points (-2, 4, -5), (1, 2, 3) is:

(A) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ (B) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ (C) $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$ (D) $\frac{-3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{-8}{\sqrt{77}}$

8. The corner points of the feasible region for a LPP are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0).
Then the minimum value of the objective function Z = $2x + 5y$ is at the point

(A) P (B) Q (C) R (D) S

9. The scalar projection of the vector $3\hat{i} - \hat{j} - 2\hat{k}$ on the vectors $\hat{i} + 2\hat{j} - 3\hat{k}$ is

(A) $\frac{7}{\sqrt{14}}$ units (B) $\frac{7}{14}$ *units* (C) $\frac{6}{13}$ *units* (D) $\frac{7}{2}$ *units*

10. The corner points of the feasible region determined by a set constraints (linear inequalities) are
P(1 ,6), Q(4, 5), R(6, 1) and S(5, 2) and the objective function is Z = $ax + 3by$ where a,b> 0. The relation between a and b such that the maximum Z occur at P and Q is

(A) a = b (B) a = 3b (C) 3a = b (D) None of these

11. Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is :

(A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$ (C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

12. $\begin{bmatrix} 3c+6 & a-d \\ a+d & 2-3b \end{bmatrix} = \begin{bmatrix} 12 & 2 \\ -8 & -4 \end{bmatrix}$ are equal, then value of ab-cd is:

(A) 4 (B) 16 (C) -4 (D) -16

13. Given two independent event A and B such that P(A) = 0.3, P(B) = 0.6 and P($A' \cap B'$) is

(A) 0.9 (B) 0.18 (C) 0.28 (D) 0.1

14. The degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 6y^5 = 0$ is

(A) 1 (B) 2 (C) 3 (D) 5

15. If $\vec{a}\,\hat{i}$ =a.($\hat{i} + \hat{j} + \hat{k}$)=1 , then $\vec{a}$ is :

(A) $\hat{k}$ (B) $\hat{i}$ (C) $\hat{j}$ (D) $\hat{i} + \hat{j} + \hat{k}$

16. The function $f(x)=\cot x$ is discontinuous on the set

(A) $\{x=n\pi, n\in Z\}$ (B) $\{x=2n\pi, n\in Z\}$

(C) $\{x=(2n+1)\frac{\pi}{2}, n\in Z\}$ (D) $\{x=\frac{n\pi}{2}, n\in Z\}$

17. If the line $\frac{x-2}{2k}=\frac{y-3}{3}=\frac{z+2}{-1}$ and $\frac{x-2}{8}=\frac{y-3}{6}=\frac{z+2}{-2}$ are parallel then value of *k* :

(A)-2 (B) $\frac{1}{2}$ (C) 2 (D) 4

18. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^3+x\cos x+\tan^5 x+1)dx$ is

(A)0 (B)2 (C) π (D)1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(A) Both (A) and (R) are true and (R) is the correct explanation of (A).
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(C) (A) is true bur (R) is false.
(D) (A) is false bur (R) is true.

19. Assertion (A) A particle moving in a straight line covers a distance of x cm. in t sec., where $x = t^3+3t^2 - 6t +18$. The velocity of the particle at the end of 3 sec. is 39 cm/sec.

Reason(R) Velocity of the particle at the end of 3 sec is $\frac{dx}{dt}$ at t = 3.

20. Assertion (A): Let R = {(a,a^3): a is a prime number < 9}, then range of R {8,27,125,343}

Reason (R): Here R = {(1,1), (2,8), (3,27), (4,64), (5,125), (6,216), (7,343), (8,512)}.

Section B

(This section comprises of very short answer type questions (VSA) of 2 marks)

21. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]+\tan^{-1}1$

OR

Prove that $3\sin^{-1}x=\sin^{-1}[3x-4x^3], x\left[-\frac{1}{2},\frac{1}{2}\right]$

22. Find the value of k for which the function f given as $f(x)=\begin{cases}\frac{1-\cos x}{2x^2} & if \quad x\neq 0\\ k & if \quad x=0\end{cases}$ is continuous at $x=0$

23. If $y\sqrt{1-x^2}+x\sqrt{1-y^2}=1$ *then prove that* $\frac{dy}{dx}=-\sqrt{\frac{1-y^2}{1-x^2}}$

OR

Show that the function f *given by* $f(x)=\tan^{-1}(\sin x+\cos x)$ is decreasing $\forall x\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

24. Evaluate $\int_{-1}^{2}\frac{|x|}{x}dx$

25. An open tank with a square base and vertical sides is to be constructed from a mettle sheet so as to be hold a given quantity of water. Show that the cost of material will be at least when depth of the tank is half of its width. If the cost is to be borne by near by settled lower income families, for whom water will be provided, what kind of value is hidden in this question?

Section C

(This section comprises of short answer type questions (SA) of 3 marks)

26. Find $\int \frac{2x}{(x^2+1)(x^2+2)}dx.$

27. Two numbers are selected at random (without replacement) from 1st seven natural numbers. If X denotes the smaller of the two numbers obtained, find the probability distribution of X. Also find mean of the distribution.

28. Find $\int \frac{1}{\sqrt{\sin^3 \cos(x-\alpha)}}dx.$

OR

Find $\int e^{\cot^{-1} x}\left(\frac{1-x+x^2}{1+x^2}\right)dx.$

29. Find the general solution of the differential equation $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$.

OR

Find the general solution of the differential equation $(x^3 + y^3)\, dy = x^2 y\, dx$.

30. Solve the following LPP graphically:

Max Z = 5x + 3y

Subject to $3x + 5y \le 15$, $5x + 2y \le 10$, and $x,y \ge 0$.

OR

Solve the following LPP graphically:

Min Z = 20x + 10y

Subject to $x + 2y \le 40$, $3x + y \ge 30$, $4x + 3y \ge 60$ and $x,y \ge 0$.

31. Find the area of following region using integration $\{(x,y): y^2 \le 2x \quad \& \quad y \ge x-4\}$

Section D

(This section comprises of long answer type questions (LA) of 5 marks)

32. Show that the height of right circular cylinder of greatest volume which can be inscribed in a right circular cone of height h and radius r is one third of the height of the cone, and the greatest volume of the cylinder is $\frac{4}{9}$ times the volume of the cone.

33. Define the relation R in the set N X N as follows:

For (a,b), (c,d) $\in$ N X N, (a,b)R(c,d) iff ad = bc. Prove that R is an equivalence relation in N X N.

OR

Let R be the set of non zero real numbers then show that $f: R \to R \quad given \quad by \quad f(x)=\frac{1}{x}$ is one one and onto.

34. If $A=\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$, $B=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$ then find AB and use it to solve the following system of equations $x-2y=3, \quad 2x-y-z=2, \quad -2y+z=3$

35. A line l passes through point (-1, 3, -2) and is perpendicular both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ *and* $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$ find the vector equation of the line l. And its distance from origin.

OR

(a) If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle 30^0 to each other, then find the area of the II^{gm} with $(\vec{a}+3\vec{b})$ *and* $(3\vec{a}+\vec{b})$ as adjacent sides.

(b) Find λ *and* μ if $(\hat{i}+3\hat{j}+9\hat{k})$ X $(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=0$

Section E

(This section comprises of three case study/passage based questions of 4 marks each with sub parts. The first two case study questions have three sub parts $((i),(ii),(iii))$ of marks 1,1,2 respectively. The third case study question has t wo sub parts of 2 marks each.)

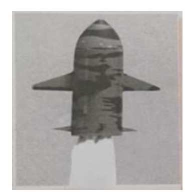
36. Read the following text and answer the following questions on the basis of the same:

The equation of motion of a missile are x = 3t, y = -4t, z = t, where the time t is given in the seconds and the distance measured in kilometers.

(i) Which of the following points lie on the path on the missile at t = 2 sec?

(ii) If the position of a rocket at a certain instant of time is (5, - 8, 10), then what will be the height of the rocket from the ground?

(iii) At what distance will be the rocket from the starting point (0,0,0) in 5 sec?



OR

What is the path of missile? and write the equation of path of missile.

37. Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by f(x)=- 0.1x^2+mx+98.6,$0\le x<12$,m being a constant, where f(x) is the temperature in ^{0}F at x days.

(i) Is the function differentiable in the interval (0,12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant m.

(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

OR

(iv) Find the points of local maximum/local minimum, if any, in the interval (0,12) as well as the points of absolute maximum/absolute minimum in the interval [0,12]. Also, find the corresponding local minimum and the absolute maximum/absolute minimum values of the function.

38. Read the following passage and answer the question given below.

A shopkeeper sells three types of flower seeds A_1,A_2,A_3. They are sold is the form of a mixture, where the proportions of these seeds are 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively.

Based on the above information:

(i) Calculate the probability that a randomly chosen seed will germinate:

(ii) Calculate the probability that the seed is of type A_2, given that a randomly chosen seed germinates.